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THE THEORY OF THE GENERALIZED MAGNUS EFFECT FOR NON-HOLONOMIC MECHANICAL SYSTEMS†

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The motion of a carriage with two wheeled pairs over a rough horizontal plane is investigated in the following cases: (1) inertial motion, (2) when there is an elastic constraint which produces a restoring moment of the forces when the axis of rotation of the leading wheeled pair deviates from the unperturbed position, and (3) when there is a small harmonic moment between the leading wheeled pair and the platform. The properties of the exact solution of the system is analysed in the first case. In the second, using the method of averaging, it is shown that for small oscillations of the leading wheeled pair with respect to the platform, after a transition process, motion of the centre of mass of the system with constant velocity, proportional to the initial amplitude of the oscillation, occurs. In the third case, the average motion of the centre of mass occurs with a constant acceleration, the value of which is estimated using the asymptotic multiscale method. © 2005 Elsevier Ltd. All rights reserved.

Recently, in relation to the problems involved in constructing microrobots, considerable attention has been devoted to the problem of finding new methods of accelerating (braking) mobile robots. In particular, the non-holonomic acceleration of mobile objects, such as skate-boards, has been discussed [1], methods of controlling the motion of different multisection crawling robots have been analysed [2, 3], the optimal control of the form of the central axis of the elastic rod, which models the motion of a snake, has been determined [4], etc.

In this paper, we investigate the particular features of the dynamics of a mobile robot, taking into account the inertial and non-holonomic properties of its construction, which takes the form of a carriage with two wheeled pairs [5–7]. The absence in this system of a traditional drive on the wheels enables its construction to be simplified considerably. The investigation of the features of the robot motion uses an analogy between the differential equations of the mobile robot and an astatic gyroscope in gimbals [8, 9].

1. THE CONSTRUCTION OF A MOBILE ROBOT

The equations of motion of a carriage. A diagram of the carriage with two wheeled pairs we are considering is shown in Fig. 1. A mobile system of coordinates x_1y_1 with origin at the point A and axis x_1 directed along the axis of symmetry of the platform is rigidly connected to the platform AB. A mobile system of coordinates x_2y_2 with origin at the point B is connected to the leading wheeled pair, which rotates about a vertical axis passing through the point B.

The position of the mechanical system described is defined by four generalized coordinates x, y, ψ, β, where x, y are the coordinates of the point A in the fixed system of coordinates XY, ψ is the "course" angle of the carriage (the angle between the X and x_1 axes), and β is the angle of rotation of the axis of the front wheeled pair with respect to the platform (the angle between the x_1 and x_2 axes).

The rear and front wheels with centres of mass at the points E_L, E_R, and F_L, F_R respectively can rotate freely about their own axes without friction. The left and right wheels on the rear and front axes have

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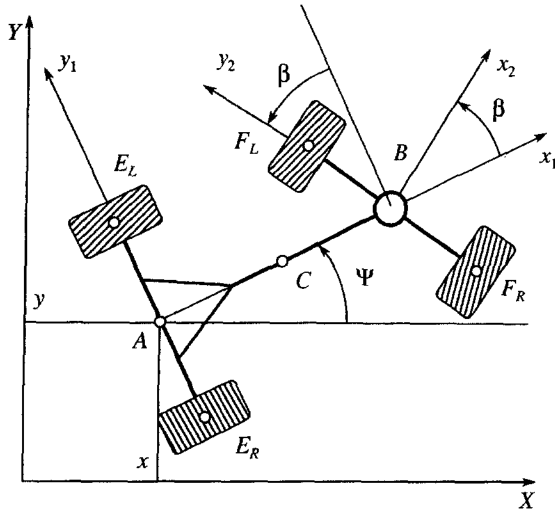

Fig. 1

the same dimensions and mass-inertia characteristics. The platform AB has a centre of mass at the point $C, AC = a$ and $AB = b$. The axis of the front wheeled pair has a centre of mass at the point B.

A moment M of a pair of forces between the platform of the carriage and the axis of the front wheeled pair is applied about the vertical axis, which is perpendicular to the plane of the figure and passes through the point B.

When there is no slip of the wheels, the projections of the velocities of the point A and B onto the y_1 and y_2 axes respectively are equal to zero, and hence the generalized coordinates and velocities satisfy the equations of non-holonomic constraints

$$-\dot{x}\sin\psi + \dot{y}\cos\psi = 0, \quad -(\dot{x}\cos\psi + \dot{y}\sin\psi)\sin\beta + \dot{\psi}b\cos\beta = 0 \tag{1.1}$$

Taking the velocity V of the point A and the relative angular velocity $\dot{\beta}$ of the front wheeled pair with respect to the platform as the pseudo-velocities, we can express the generalized velocities in terms of the pseudo-velocities.

$$\dot{x} = V\cos\psi, \quad \dot{y} = V\sin\psi, \quad \dot{\psi} = \frac{V}{b}\mathrm{tg}\beta, \quad \dot{\beta} = \dot{\beta} \tag{1.2}$$

Carrying out appropriate calculations to determines the accelerations of the centres of mass and angular acceleration of the parts of the system, we obtain Appell's function – the "acceleration energy" $S = S(\dot{V}, \ddot{\beta}, V, \dot{\beta}, x, y, \psi, \beta)$. Omitting the terms that do not depend on the accelerations, we finally obtain the following expression

$$S = \frac{1}{2}(m + m_0\mathrm{tg}^2\beta)\dot{V}^2 + \frac{I_2}{b}\mathrm{tg}\beta\dot{V}\ddot{\beta} + \frac{1}{2}I_2\ddot{\beta}^2 + \frac{m_0\mathrm{tg}\beta}{\cos^2\beta}V\dot{V}\dot{\beta} + \frac{I_2}{b\cos^2\beta}V\dot{\beta}\ddot{\beta}$$

Here

$$m = 3m_E + 3m_F + m_C + m_B, \quad I_2 = I_B + m_F(r_F^2/2 + 3l_F^2)$$

$$m_0 = b^{-2}(I_2 + I_C + m_E r_E^2/2 + m_C a^2 + m_B b^2 + 3m_F b^2 + 3m_E l_E^2)$$

$a = AC$ is the distance from the rear axis to the centre of mass of the platform, $b = AB$ is the distance between the rear and front axes, $l_E = AE_L = AE_R$ is the half-length of the rear axis, $l_F = AF_L = AF_R$ is the half-length of the front axis, r_E and r_F are the radius of the rear and front wheels respectively, m_C is the mass of the platform, I_C is the moment of inertia of the platform about the point C, m_E is the mass of the rear wheel, $I_E = m_E r_E^2/2$ is the moment of inertia of the rear wheel about its axis of rotation, m_F is the mass of the front wheel, $I_F = m_F r_F^2/2$ is the moment of inertia of the front wheel about its axis of rotation, m_B is the mass of the front axis (without the wheels), and I_B is the moment of inertia of the front axis about the point B.

We will take Appell's equations in pseudo-velocities [5]

$$\frac{\partial S}{\partial \dot{V}} = Q_V, \quad \frac{\partial S}{\partial \ddot{\beta}} = Q_\beta$$

as the initial equations of the carriage motion.

Here Q_V and Q_β are generalized forces, where, in the case considered here, $Q_V = 0$, $Q_\beta = M$ and M is the moment of the pair of forces applied to the front wheeled pair (in this case a moment $-M$ will be applied to the platform AB).

After differentiating Appell's function we obtain a sixth-order system of non-linear differential equations.

$$\dot{x} = V\cos\psi, \quad \dot{y} = V\sin\psi, \quad \dot{\psi} = \frac{V}{b}\operatorname{tg}\beta$$
$$\mu_0(\beta)\dot{V} + \frac{I_2\operatorname{tg}\beta}{b}\ddot{\beta} = -\frac{m_0\operatorname{tg}\beta}{\cos^2\beta}V\dot{\beta} \tag{1.3}$$
$$\frac{I_2\operatorname{tg}\beta}{b}\dot{V} + I_2\ddot{\beta} = M - \frac{I_2}{b\cos^2\beta}V\dot{\beta}$$

where $\mu_0(\beta) = m + m_0\operatorname{tg}^2\beta$.

The last two equations of system (1.3) are always solvable for the higher derivatives. Here, if we put $m_1 = m_0 - I_2/b^2$, we obtain a third-order system of two non-linear differential equations, which can be separated from the kinematic equations in the case when the moment M is independent of the generalized coordinates x, y, ψ,

$$\dot{V} = -\frac{\operatorname{tg}\beta}{b\mu_1(\beta)}M - \frac{m_1\operatorname{tg}\beta}{\mu_1(\beta)\cos^2\beta}V\dot{\beta}, \quad \ddot{\beta} = \frac{\mu_0(\beta)}{I_2\mu_1(\beta)}M - \frac{m}{b\mu_1(\beta)\cos^2\beta}V\dot{\beta} \tag{1.4}$$

where $\mu_1(\beta) = m + m_1\operatorname{tg}^2\beta$.

Below we will investigate system (1.4) in the following cases:

(1) $M = 0$ – free motion of the carriage, for which we can construct an exact solution of system (1.4);

(2) $M = -K\beta$ – there is elastic constraint between the front and rear pairs of wheels and the platform;

(3) $M = M_0\cos\nu t$ there is an internal active periodic moment with amplitude M_0 and frequency ν.

Remark 1. The problem of integrating Eqs (1.4) of the free motion of the carriage was formally reduced to quadratures in [6]. It was asserted [6, p. 117], that the solution obtained describes a system with self-orienting front wheels. However, the analysis of the exact solution of Eqs (1.4) carried out below shows that the carriage considered, unlike robots with front rollers [10], generally does not possess the property of self-orientation of the front wheels leading to rectilinear translational motion of the platform.

2. THE INERTIAL FREE MOTION OF THE SYSTEM

When there is no moment ($M = 0$) we obtain the following system of non-linear differential equations from system (1.4)

$$\dot{\beta} = \omega, \quad \dot{\omega} = -\frac{m}{b\mu_1(\beta)\cos^2\beta}V\omega, \quad \dot{V} = -\frac{m_1\operatorname{tg}\beta}{\mu_1(\beta)\cos^2\beta}V\omega \tag{2.1}$$

System (2.1) has an entire plane of equilibrium states

$$\omega = 0, \quad V = \text{const}, \quad \beta = \text{const}$$

When $M = 0$, from the last equation of (1.3) we obtain the integral of the angular momentum of the front wheeled pair about the vertical axis

$$I_{22}\left(\frac{\operatorname{tg}\beta}{b}V + \omega\right) = K_z = \text{const} \tag{2.2}$$

The last equation of system (2.1) is an equation with separable variables, from which we obtain the first integral (the energy integral)

$$(m + m_1 \operatorname{tg}^2\beta)V^2 = \text{const} = 2T_0 \tag{2.3}$$

Eliminating $\operatorname{tg}\beta$ from Eqs (2.2) and (2.3), we obtain

$$mV^2 + m_1 b^2 (K_z/I_2 - \omega)^2 = 2T_0 \tag{2.4}$$

Hence, in the three-dimensional phase space β, ω, V of system (1.3), the projection of the phase trajectory onto the ω, V plane is the ellipse (2.4), the centre of which is at the point $V = 0$, $\omega = K_z/I_2$. The semiaxes of this ellipse are respectively $\sqrt{2T_0/m}$, $\sqrt{2T_0/(m_1 b^2)}$.

If the condition

$$I_A - m_C b^2 = m_E(3b^2 - 3l_E^2 - r_E^2/2), \quad I_A = I_C + m_C a^2 \tag{2.5}$$

is satisfied, which can be achieved by an appropriate choice of the geometrical parameters and the distribution of the masses of the platform and of the front wheels of the carriage, and if the initial conditions satisfy the relation

$$\sqrt{\frac{2T_0}{m_1 b^2}} - \frac{K_z}{I_2} > 0 \tag{2.6}$$

the solution of Eqs (2.1) can be written in the form

$$\operatorname{tg}\beta(t) = \frac{1}{2}\frac{(c_2 + c_1)\xi(t) - c_2 + c_1}{\xi(t)\sqrt{c_2^2 - c_1^2}}, \quad V(t) = \pm\sqrt{\frac{2T_0}{\mu_1(t)}}$$

$$\xi(t) = \operatorname{th}\left(\frac{1}{2}\sqrt{c_2^2 - c_1^2}(t + u_0)\right), \quad c_1 = \frac{K_z}{I_2}, \quad c_2 = \sqrt{\frac{2T_0}{m_1 b^2}}$$

Note that condition (2.5) is analogous to the limitation imposed on the mass distribution in an astatic gyroscope in gimbals, considered in [11], in which, when integrating the equations of motion, the hyperelliptic integrals are reduced to elliptic integrals.

When condition (2.6) is satisfied, the ellipse (2.4) intersects the plane $\omega = 0$ of the equilibrium states of system (2.1). The corresponding phase trajectory asymptotically approaches the point

$$V^* = \sqrt{\frac{2I_2^2 T_0 - m_1 b^2 K_z^2}{m I_2^2}}, \quad \operatorname{tg}\beta^* = \pm\sqrt{\frac{mb^2 K_z^2}{2I_2^2 T_0 - m_0 b^2 K_z^2}} \tag{2.7}$$

Stationary solution (2.7) represents the uniform rotation of the platform around a vertical axis. The point A in this case moves with constant velocity V^* along a circle of radius $b \operatorname{ctg}\beta^*$.

If the initial conditions for the differential equations are such that

$$\sqrt{\frac{2T_0}{m_3 b^2}} < \left|\frac{K_z}{I_2}\right| \tag{2.8}$$

system (1.4) will not have stationary solutions. Possible types of trajectories of the point A, obtained by numerical integration of system (1.3), are shown in Fig. 2 in the case when inequality (2.8) is satisfied (a) and when the system reaches a steady state (2.7) (b).

3. THE MOTION OF THE CARRIAGE WHEN THERE IS AN ELASTIC MOMENT

We will consider the case when the moment of the pair of forces is proportional to the angle of rotation β of the front wheeled pair of wheels with respect to the platform

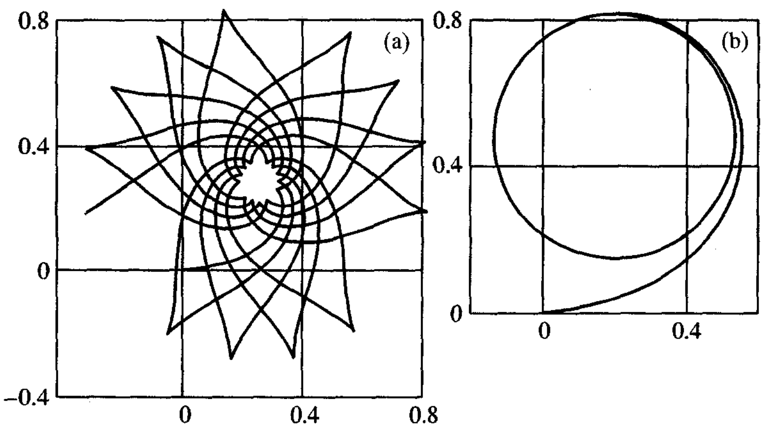

Fig. 2

$$M = -K\beta \tag{3.1}$$

Here K is the stiffness of the corresponding spring. Equations (1.4) can then be written in the form

$$\dot{\beta} = \omega, \quad \dot{\omega} = -\frac{K\beta\mu_0(\beta)}{I_2\mu_1(\beta)} - \frac{m}{b\mu_1(\beta)\cos^2\beta}V\omega, \quad \dot{V} = \frac{K\beta\,\mathrm{tg}\,\beta}{b\mu_1(\beta)} - \frac{m_1\,\mathrm{tg}\,\beta}{\mu_1(\beta)\cos^2\beta}V\omega \tag{3.2}$$

Remark 2. Chaplygin's reducing factor method was used in [6] to analyse this case, from which it was not possible "to extract any practical recommendations" or to understand the pattern of the motion, and hence a fairly lengthy qualitative analysis of the energy integral was carried out in [6]. The asymptotic method used below enables the motion of the carriage to be described almost immediately.

Changing to dimensionless variables

$$\beta = \varepsilon x_1, \quad \omega = \varepsilon\Omega x_2, \quad V = \varepsilon\Omega b x_3, \quad K = jmb^2\Omega^2, \quad I_2 = jmb^2$$

and introducing the dimensionless time $\tau = \Omega t$ ($\Omega = \sqrt{K/I_2}$), after expanding the right-hand sides of Eqs (3.2) in series in the small parameter ε, we obtain

$$\begin{aligned}
x_1' &= x_2 \\
x_2' &= -x_1 - \varepsilon x_2 x_3 + \varepsilon^2\frac{m_1 - m_0}{m}x_1^3 + \varepsilon^3\frac{m_1 - m}{m}x_1^2 x_2 x_3 + \varepsilon^4\frac{(2m - 3m_1)(m_1 - m_0)}{3m^2}x_1^5 - \\
&-\varepsilon^5\left(\frac{2}{3} - \frac{5m_1}{3m} + \frac{m_1^2}{m^2}\right)x_1^4 x_2 x_3 \\
x_3' &= \varepsilon j x_1^2 - \varepsilon^2\frac{m_1}{m}x_1 x_2 x_3 - \varepsilon^3 j\frac{3m_1 - m}{3m}x_1^4 + \varepsilon^4\frac{m_1(3m_1 - 4m)}{3m^2}x_1^3 x_2 x_3 + \\
&+\varepsilon^5 j\left(\frac{2}{15} - \frac{m_1}{m} + \frac{m_1^2}{m^2}\right)x_1^6
\end{aligned} \tag{3.3}$$

Equations (3.3) can be reduced to the standard form of the asymptotic method of averaging by changing the variables

$$x_1 = A\cos\varphi, \quad x_2 = -A\sin\varphi \tag{3.4}$$

where a is the amplitude and φ is the phase. Averaging the right-hand sides of the equations for the slow variables a and x_3 with respect to the fast variable φ, we obtain the following system of average equations

$$A' = -\frac{\varepsilon}{2}Ax_3, \quad x_3' = \frac{\varepsilon}{2}jA^2 \tag{3.5}$$

The first integral of Eqs (3.5) has the form

$$jA^2 + x_3^2 = \text{const} \tag{3.6}$$

If, at the initial instant, the system is in a state of rest ($a(0) = a_0, x_3(0) = 0$), it follows from (3.5) that its solution approaches the point $a(\infty) = 0, x_3(\infty) = a_0\sqrt{j}$. Reverting to dimensional variables, we conclude that the centre of mass of the system will tend to rectilinear motion with constant velocity equal to $\beta_0\sqrt{K/m}$.

4. ASYMPTOTIC SOLUTION OF THE EQUATIONS OF MOTION WHEN THERE IS A PERIODIC MOMENT. VIBRATIONAL ACCELERATION OF THE CARRIAGE

When there is a periodic moment, we take the first integrals of the unperturbed problem, obtained in Section 2, as the new variables and, instead of the variables β, $\dot{\beta}$, V, we introduce the variables z, T_0, K_z related to the old variables by the formulae

$$z = \operatorname{tg}\beta, \quad T_0 = \frac{1}{2}(m + m_1\operatorname{tg}^2\beta)V^2, \quad K_z = I_2\left(\frac{V}{b}\operatorname{tg}\beta + \dot{\beta}\right) \tag{4.1}$$

The inverse transformation from the new variables to the old ones has the form

$$\beta = \operatorname{arctg} z, \quad \dot{\beta} = \frac{K_z}{I_2} - \frac{z}{b}\sqrt{\frac{2T_0}{m + m_1z^2}}, \quad V = \sqrt{\frac{2T_0}{m + m_1z^2}} \tag{4.2}$$

The new variables satisfy the equations

$$\begin{gathered} \frac{dz}{dt} = \left(\frac{K_z}{I_2} - \frac{z}{b}\sqrt{\frac{2T_0}{m + m_1z^2}}\right)(1 + z^2) \\ \frac{dK_z}{dt} = M, \quad \frac{dT_0}{dt} = -\frac{M}{b}z\sqrt{\frac{2T_0}{m + m_1z^2}} \end{gathered} \tag{4.3}$$

If the moment applied to the front wheeled pair is small, to investigate system (4.3) we can use the asymptotic method because in system (4.3) the variables K_z and T_0 are slow variables while z is a fast variable.

To reduce the complexity of the formulae, we will confine ourselves to the special case when

$$K_z = K_0\sin\nu t, \quad M = K_0\nu\cos\nu t, \quad K_0 = \text{const}, \quad \nu = \text{const}$$

System (4.3) then takes the form

$$\begin{gathered} \frac{dz}{dt} = \left(\frac{K_0\sin\nu t}{I_2} - \frac{z}{b}\sqrt{\frac{2T_0}{m + m_1z^2}}\right)(1 + z^2) \\ \frac{dT_0}{dt} = -\frac{K_0z\nu\cos\nu t}{b}\sqrt{\frac{2T_0}{m + m_1z^2}} \end{gathered} \tag{4.4}$$

After making the replacement of variables

$$\begin{gathered} T_0 = T^*y^2, \quad K_0 = \varepsilon I_2\nu, \quad t = \frac{\tau}{\nu\varepsilon}, \quad z = \frac{2\varepsilon z_1}{1 - \varepsilon^2 z_1^2} \\ T^* = \frac{mb^2\nu^2}{2}, \quad I_2 = \kappa\frac{mb^2}{2}, \quad m_1 = \mu m \end{gathered} \tag{4.5}$$

Eqs (4.4), for small values of the parameter ε, become a singularly perturbed system of non-linear differential equations

$$\varepsilon\frac{dz_1}{d\tau} = -\frac{z_1(1+\varepsilon^2 z_1^2)y}{\Delta(z_1)} + \frac{1}{2}(1+\varepsilon^2 z_1^2)\sin\frac{\tau}{\varepsilon}, \quad \frac{dy}{d\tau} = -\frac{\varepsilon\kappa z_1}{\Delta(z_1)}\cos\frac{\tau}{\varepsilon} \tag{4.6}$$

$$\Delta(z_1) = \sqrt{(1-\varepsilon^2 z_1^2)^2 + 4\mu_1\varepsilon^2 z_1^2}$$

Here z_1 is the "fast" variable, proportional to tg(β/2), y is the "slow" variable, and ε and κ are dimensionless parameters

$$\varepsilon = \frac{K_0}{I_2 v}, \quad \kappa = \frac{2I_2}{mb^2} \tag{4.7}$$

In order to avoid singularities of τ/ε under the sign of the trigonometric function, we introduce the following notation

$$\sin\frac{\tau}{\varepsilon} = z_2, \quad \cos\frac{\tau}{\varepsilon} = z_3 \tag{4.8}$$

and supplement system (4.6) with two differential equations for z_2 and z_3 with initial conditions

$$z_2(0) = 0, \quad z_3(0) = 1$$

We finally obtain a singularly perturbed system of fourth-order differential equations

$$\frac{dy}{d\tau} = -\frac{\varepsilon\kappa z_1 z_3}{\Delta(z_1)}, \quad \varepsilon\frac{dz_1}{d\tau} = -z_1 y\frac{1+\varepsilon^2 z_1^2}{\Delta(z_1)} + \frac{1}{2}(1+\varepsilon^2 z_1^2)z_2, \quad \varepsilon\frac{dz_2}{d\tau} = z_3, \quad \varepsilon\frac{dz_3}{d\tau} = -z_2 \tag{4.9}$$

$$y|_{\tau=0} = y^0, \quad \mathbf{z}|_{\tau=0} = \mathbf{z}^0$$

Note that, apart from quantities of the order ε^6, the first two equation of the system can be written in the form

$$\begin{aligned} \frac{dy}{d\tau} &= -\varepsilon\kappa z_1 z_3(1+\varepsilon^2 z_1^2(1-2\mu_1) + \varepsilon^4 z_1^4(1-6\mu_1+6\mu_1^2)+\ldots) \\ \varepsilon\frac{dz_1}{d\tau} &= -z_1 y(1+2\varepsilon^2 z_1^2(1-\mu_1) + 2\varepsilon^4 z_1^4(1-4\mu_1+3\mu_1^2)+\ldots) + \frac{1}{2}(1+\varepsilon^2 z_1^2)z_2 \end{aligned} \tag{4.10}$$

The asymptotic solution of system (4.9) can be constructed using the "multiscale method", according to which the system is replaced by a system of partial differential equation [12, pp. 43–52]. This solution, constructed for the "slow" variable y, apart from quantities of the order of ε^2, contains a term that increases linearly with time

$$y(\tau) = y^0 + \frac{\varepsilon\kappa\tau}{4(1+y^{02})} + \ldots \tag{4.11}$$

Correspondingly, we have the following estimates for the kinetic energy and velocity of the point A

$$T_0 = \frac{M_A^4}{8mb^2 I_2^2 v^4}t^2, \quad V \approx \frac{M_A^2}{2mbI_2 v^2}t \tag{4.12}$$

from which it follows that it is possible for the robot to accelerate when there is a periodic moment between the platform and the front wheeled pair.

Here we point out a certain analogy between the effect represented by (4.11) and the Magnus effect [8] for a balanced gyroscope in gimbals, when, if the axis of symmetry of the rotor vibrates, a systematic rotation of the outer ring of the gimbals occurs. The direction of this rotation is determined by the sign of the angle of rotation β of the inner ring of the gimbals. According to formula (4.11) the direction

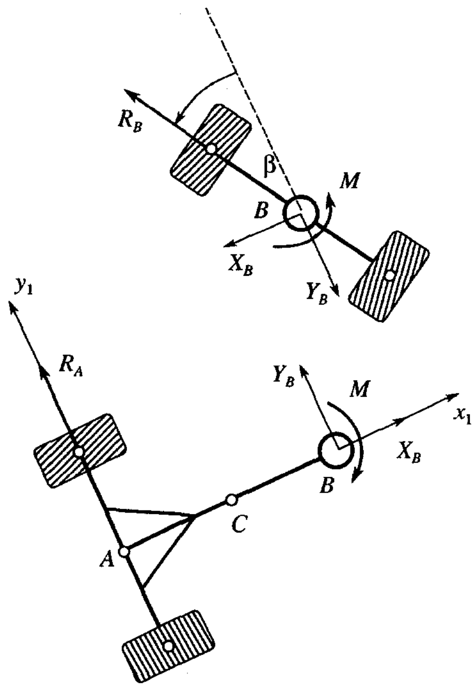

Fig. 3

of motion of the carriage is determined by the sign of b, i.e. by the direction of the vector AB, drawn from the middle of the rear wheeled pair to the point of the hinged fastening of the front wheeled pair.

5. AN ESTIMATE OF THE CONDITIONS OF REALIZABILITY OF NON-HOLONOMIC CONSTRAINTS

Appell's equations (1.3) were obtained when the conditions for non-holonomic constraints (1.1) were satisfied, i.e. when there was no slip of the wheels of the carriage on the horizontal surface. However, if the friction forces at the point where the wheels are in contact with the surface exceed the limit value of the Coulomb dry-friction forces, the wheels begin to slide and the motion of the system will not be described by Eqs (1.3). Hence, we will determine the friction forces at the points where the wheels are in contact with the surface, which enables us to estimate the characteristics of the construction and the parameters of the "accelerating" moment M (its amplitude and frequency), for which sliding begins.

In deriving the required relations we will use the general theorems of dynamics, which are written for two subsystems (the platform AB and the front wheeled pair B), shown in Fig. 3. Here, for simplicity, we will confine ourselves to the case of weightless wheels. To estimate the realizability of non-holonomic constraints we will use Coulomb's axiom

$$|R_A| \le fN_A, \quad |R_B| \le fN_B \tag{5.1}$$

where f is the coefficient of dry friction, and N_A and N_B are the reactions of the support to the surface.

After appropriate calculations, we obtain the following system of inequalities

$$\begin{aligned} &\left|(m_C a + m_B b)\dot{\Omega} + m_C \frac{(b-a)V^2 \operatorname{tg}\beta}{b^2} + m\dot{V}\operatorname{ctg}\beta\right| \le f m_C g\left(\frac{b-a}{b}\right) \\ &\left|(m_C a + m_B b)\frac{V^2 \operatorname{tg}\beta}{b^2 \cos\beta} - \frac{m\dot{V}}{\sin\beta}\right| \le fg\left(\frac{m_C a + m_B b}{b}\right) \end{aligned} \tag{5.2}$$

which defines the domain of variation of the variables V, $\dot{V}$, β, $\dot{\Omega}$, in which the constraint equations (1.1) are satisfied. Outside this domain sliding begins.

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